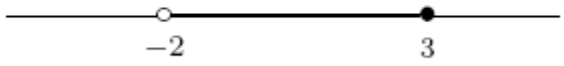
Inequalities

10.1 *Number line graphs*. Observe the following conventions, which may already be familiar:

- To indicate an interval on the number line, thicken that part of the number line
- To indicate that an endpoint of an interval is included, place a solid dot on the number.
- To indicate that an endpoint is not included, place an open circle on the number.

For example, the diagram illustrates those numbers that are greater than -2 and less than or equal to 3.



Draw a number line for each of the following and indicate the numbers described:

a) All numbers that are exactly two units from 5.
b) All numbers that are more than two units from 5.
c) All numbers that are greater than -1 and less than or equal to 7.
d) All numbers that are less than four units from zero.

10.7 On a number line, graph all numbers that are closer to 5 than they are to 8.

11.10 On a number line, graph a number that is twice as far from 5 as it is from 8. How many such numbers are there?

12.1 Intervals on a number line are often described using symbols < (less than), > (greater than), $\leq$ (less than or equal to), and $\geq$ (greater than or equal to). As you graph the following inequalities on a number line, remember the conventions regarding the use of the dot • and the circle for included and excluded endpoints, respectively:

a) $x < 5$ b) $x \geq -6$ c) $-12 \geq x$ d) $4 < x < 8$ e) $x < -3$ or $7 \leq x$

12.8 Graph on a number line the intervals described below:

a) All numbers that are greater than 1 or less than -3.
b) All numbers that are greater than -5 and less than or equal to 4.
c) All numbers whose squares are greater than or equal to 1.

12.9 Use mathematical notation to represent the intervals described below.

a) All numbers that are greater than 1 or less than -3.
b) All numbers that are greater than -5 and less than or equal to 4.
c) All numbers whose squares are greater than or equal to 1.

13.8 Place a common mathematical symbol between the numerals 2 and 3, so as to produce a number that lies between 2 and 3 on a number line.

15.2 Graph on a number line the intervals corresponding to these two signs on the highway.
   a) The maximum speed is 65 mph and the minimum speed is 45 mph.
   b) The maximum speed is 55 mph.

16.1 The statement "x is between 13 and 23" defines an interval using two simultaneous inequalities: $13 < x$ and $x < 23$. The statement "x is not between 13 and 23" also uses two inequalities, but they are non-simultaneous: $x \leq 13$ or $23 \leq x$. Graph these two examples on a number line. Notice that there is a common form $13 < x < 23$ for only one of them.

17.1 Chandler was given $75 for a birthday present. This present, along with earnings from a summer job, is being set aside for a mountain bike. The job pays $6 per hour, and the bike costs $345. To be able to buy the bike, how many hours does Chandler need to work?

17.2 (continuation) Let h be the number of hours that Chandler works. What quantity is represented by the expression 6h? What quantity is represented by the expression 6h + 75?
   a) Graph the solutions to the inequality $6h + 75 \geq 345$ on a number line.
   b) Graph the solutions to the inequality $6h + 75 < 345$ on a number line.
   What do the solutions to the inequality $6h + 75 \geq 345$ signify?

17.7 Solve the inequality $3 - x > 5$ using only the operations of addition and subtraction. Is $x = 0$ a solution to the inequality?

20.5 Pat and Kim are solving the inequality $132 - 4x \leq 36$. Each begins by subtracting 132 from both sides to get $-4x \leq -96$, and then each divides both sides by –4. Pat gets $x \leq 24$ and Kim gets x u 24, however. Always happy to offer advice, Alex now suggests to Pat and Kim that answers to inequalities can often be checked by substituting $x = 0$ into both the original inequality and the answer. What do you think of this advice? Graph each of these answers on a number line. How do the results of this question relate to the flooding of the Exeter River?

20.6 (continuation) After hearing Alex's suggestion about using a test value to check an inequality, Wes suggests that the problem could have been done by solving the equation $132 - 4x = 36$ first. Complete the reasoning behind this strategy.

20.7 (continuation) Deniz, who has been keeping quiet during the discussion, remarks, "The only really tricky thing about inequalities is when you try to multiply them or divide them by negative numbers, but this kind of step can be avoided altogether. Wes just told us one way to avoid it, and there is another way, too." Explain this remark by Deniz.

20.9 Solve the inequality for x: $2(1 - 3x) - (x - 5) > 1$

23.8 Find the value of x that makes $0.1x + 0.25(102 - x) = 17.10$ true.

23.10 Find all the values of x that make $0.1x + 0.25(102 - x) < 17.10$ true.

24.6 Find and graph the solution sets for the related questions:

a) $46 - 3(x + 10) = 5x + 20$
b) $46 - 3(x + 10) < 5x + 20$
c) $46 - 3(x + 10) > 5x + 20$

28.1 Given that $48 \leq n \leq 1296$ and $24 \leq d \leq 36$, what are the largest and smallest values that the expression n/d can possibly have? Write your answer *smalles*t < n/d < *largest*.

28.5 The equation $5x - 8y = 20$ expresses a linear relationship between x and y. The point (15,7) is either on the graph of the line, above it, or below it. Which? How do you know?

30.12 Jay thinks that the inequality $k < 3$ implies the inequality $k^2 < 9$, but Val thinks otherwise. Who is right, and why?

37.2 Lee's pocket change consists of x quarters and y dimes. Put a dot on every lattice point (x,y) that signifies that Lee has exactly one dollar of pocket change. What equation describes the line that passes through these points? Notice that it does not make sense to connect the dots in this context, because x and y are discrete variables, whose values are limited to integers.

37.3 (continuation) Put a dot on every lattice point (x,y) that signifies that Lee has at most one dollar in pocket change. How many such dots are there? What is the relationship between Lee's change situation and the inequality $0.25 + 0.10y \leq 1.00$?

37.4 Write two inequalities that stipulate that Lee cannot have fewer than zero quarters or fewer than zero dimes.

38.9 Brett is holding three quarters and five dimes. Does Brett have more than one dollar or less than one dollar? Does the point (3,5) lie above or below the line $0.25x + 0.10y = 1.00$?

39.5 Graph the equation $2x + 3y = 6$. Now graph the inequality $2x + 3y \leq 6$ by shading all points (x,y) that fit it. Notice that this means shading all the points on one side of the line you drew. Which side? Use a test point like (0,0) to decide.

40.4 Sandy's first four test scores this term are 73, 87, 81, and 76. To have at least a B test grade, Sandy needs to average at least 80 on the five term tests (which count equally). Let t represent Sandy's score on the fifth test, and write an inequality that describes the range of t-values that will meet Sandy's goal.

40.5 Shade the points in a plane whose x-coordinates are greater than their y-coordinates. Write an inequality that describes these points.